A Solution to the So-Called 3-Generations Problem of Elementary Particles Or: Why are There 3 masses for Charged Leptons, Neutrons and Quarks? Starting Point this time: The Combined Metric Klein-Gordon + Dirac Equation

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The Three Generations of Particles, by Livia Schwarzer

Theory: The Ricci Scalar for a Scaled Metric

In [1, 2, 3] (and quite some other papers by this author) it was demonstrated how we can find a direct extraction of the Klein-Gordon, the Schrödinger and the Dirac equation from the Ricci scalar R^* of a modified metric of the kind $G_{\alpha\beta}=F[f]*g_{\alpha\beta}$. Namely: With a yet arbitrary scalar function F[f] the corresponding modified Ricci scalar R^* reads:

$$R^*=\frac{1}{F[f]^3}\cdot\left(\begin{array}{c}\left(C_{N1}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2-C_{N2}\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\overbrace{\left(\tilde{\nabla}_g f\right)^2}^{=f_{,\alpha}g^{\alpha\beta}f_{,\beta}}\\ -C_{N2}\cdot F[f]\cdot\frac{\partial F[f]}{\partial f}\cdot\Delta_g f\end{array}\right). \quad (1)$$

$$\xrightarrow{n=4}=\frac{1}{F[f]^3}\cdot\left(\left(\frac{3}{2}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2-3\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\left(\tilde{\nabla}_g f\right)^2-3\cdot F[f]\cdot\frac{\partial F[f]}{\partial f}\cdot\Delta_g f\right)$$

Thereby the C_{N1} and C_{N2} are constants depending on the number of dimensions n of the space-time being considered. For illustration we also gave the case n=4 (n=number of dimensions). Thereby f is to be understood as a function of the coordinates $f=f[x_0, x_1, x_2, \ldots]$. It was shown in [1 - 6] (especially [2]) that we can write:

$$\begin{array}{c}C_{N1}=-\frac{3}{2}+\frac{7}{4}\cdot n-\frac{1}{4}\cdot n^2\\ C_{N2}=n-1\end{array}, \quad (2)$$

Demanding certain conditions for the function F[f] and / or f then gives us Dirac or Klein-Gordon equations [1, 2, 3]. Thus, when demanding f to be a Laplace function, we obtain from (1):

$$\begin{array}{l}R^*=\frac{1}{F[f]^3}\cdot\left(\left(C_{N1}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2-C_{N2}\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\overbrace{\left(\tilde{\nabla}_g f\right)^2}^{=f_{,\alpha}g^{\alpha\beta}f_{,\beta}}\right),\\ \xrightarrow{n=4}=\frac{1}{F[f]^3}\cdot\left(\left(\frac{3}{2}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2-3\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\left(\tilde{\nabla}_g f\right)^2\right)\end{array} \quad (3)$$

which – so it was shown in [1, 2, 3] – gives the metric equivalent to the Dirac equation. It was also shown in [1, 2] how this gives the classical Dirac equation in flat space Minkowski metrics.

Finally – Klein Gordon + Dirac: A Most Simple Explanation for the 3-Generations Problem

This time we will not try to get rid of one of the two differential operators in equation (1), but assume them to have eigen solutions with eigenvalues A and B as shown in [2] or our earlier posts here (see "General F[f] - here Nabla and Laplace with Eigenvalues.pdf"). Now, just as in the most recent post, we apply the following simple linear form for F[f] via F[f]=(f+C_f/M). This gives us from equation (1), with time without the assumption of f being a Laplace solution, the following result:

$$\begin{aligned}R^{*}&=\frac{(1-n)}{4\cdot\left(f+\frac{C_f}{M}\right)^3}\cdot\left((6-n)\cdot f_{,\alpha}g^{\alpha\beta}f_{,\beta}+4\cdot\left(f+\frac{C_f}{M}\right)\cdot\Delta f\right)\\&=f\cdot\frac{(1-n)}{4\cdot\left(f+\frac{C_f}{M}\right)^3}\cdot\left(A\cdot(6-n)\cdot f+4\cdot B\cdot\left(f+\frac{C_f}{M}\right)\right)\end{aligned},\qquad(4)$$

Now we follow the path outlined in the previous post and algebraically solve the equation above. In order to make things easier with respect to the 3-generations problem, we substitute as follows:

$$\Psi=M\cdot f;\, R^{**}=4\cdot\frac{R^{*}}{M};\, B_s=4\cdot(1-n)\cdot B;\, A_s=A\cdot(1-n)\cdot(n-6)$$

$$\begin{aligned}&\Rightarrow R^{*}=\frac{\Psi}{M}\cdot\frac{(1-n)}{4\cdot\left(\frac{\Psi}{M}+\frac{C_f}{M}\right)^3}\cdot\left(A\cdot(6-n)\cdot\frac{\Psi}{M}+4\cdot B\cdot\left(\frac{\Psi}{M}+\frac{C_f}{M}\right)\right)\qquad,\\&\Rightarrow 4\cdot R^{*}\cdot\left(\frac{\Psi}{M}+\frac{C_f}{M}\right)^3=\frac{\Psi}{M}\cdot(1-n)\cdot\left(A\cdot(6-n)\cdot\frac{\Psi}{M}+4\cdot B\cdot\left(\frac{\Psi}{M}+\frac{C_f}{M}\right)\right)\\&\Rightarrow R^{**}\cdot(\Psi+C_f)^3=4\cdot\frac{R^{*}}{M}\cdot(\Psi+C_f)^3=\Psi\cdot(A_s\cdot\Psi+B_s\cdot(\Psi+C_f))\end{aligned}\qquad(5)$$

which gives – after expansion and division by R^{**} – from (4):

$$(C_f)^3-\frac{B_s\cdot C_f}{R^{**}}+\left(3\cdot(C_f)^2-\frac{A_s}{R^{**}}-\frac{B_s}{R^{**}}\right)\cdot\Psi+3\cdot C_f\cdot\Psi^2+\Psi^3=0,\qquad(6)$$

Again, we point out that for here and now (for the reason of simplicity and brevity mainly) we do not need to care about potential inner vector characters of the mass, the function f and the operator terms. For the moment we just assume that this does not have any influence on the 3-generations problem we want to consider here. The proof for this can easily be obtained by applying $f=h_\alpha q^\alpha$ in all derivations below. It will not change the principle results with regards to the 3-generation or 3-masses problem.

Equation (6) is a polynomial of third order and it can have three solutions. The general solution to a three-order polynomial could be given via the following product form:

$$\begin{aligned}&(\Psi-\Psi_1)\cdot(\Psi-\Psi_2)\cdot(\Psi-\Psi_3)=\\&\Psi^3-\Psi^2\cdot(\Psi_1+\Psi_2+\Psi_3)+\Psi\cdot(\Psi_1\Psi_2+\Psi_1\Psi_3+\Psi_2\Psi_3)-\Psi_1\Psi_2\Psi_3\end{aligned}.\qquad(7)$$

Comparing the latter with (6) gives us:

$$
\begin{aligned}
3\cdot C_f &= -\left(\Psi_1+\Psi_2+\Psi_3\right)\\
3\cdot\left(C_f\right)^2-\frac{A_s}{R^{**}}-\frac{B_s}{R^{**}} &= \Psi_1\Psi_2+\Psi_1\Psi_3+\Psi_2\Psi_3\,.\\
\left(C_f\right)^3-\frac{B_s\cdot C_f}{R^{**}} &= -\Psi_1\Psi_2\Psi_3
\end{aligned}
\tag{8}
$$

Thus, we have obtained the three generations of quantum gravity solutions to the combined mass-times-f-function, given via $\Psi\equiv M\cdot f\equiv\frac{m\cdot c}{\hbar}\cdot f$, as functions or dependencies of the Ricci scalar R* of the quantum-gravity (just scalar) variated metric $G_{\alpha\beta}=F[f]*g_{\alpha\beta}$, the mass-values $M\equiv\frac{m\cdot c}{\hbar}$ and a constant C_f.

Please note: As the expressions $\Psi_1+\Psi_2+\Psi_3,\ \Psi_1\Psi_2+\Psi_1\Psi_3+\Psi_2\Psi_3,\ \Psi_1\Psi_2\Psi_3$ on the right-hand side in (8) should be just constants and thus, could not depend on the various masses M_i, we have to demand the Ricci-scalar R^* to be directly connected with the M_i via the two constants $const_1$ and $const_2$:

$$
\text{I)}\quad\begin{cases}
\frac{A_s}{R^{**}}+\frac{B_s}{R^{**}}=const_1=\frac{(1-n)}{4\cdot R^*}\cdot M\cdot\left(A\cdot(n-6)+4\cdot B\right)\\
\Rightarrow\quad R^*=\frac{(1-n)}{4}\cdot M\cdot\frac{\left(A\cdot(n-6)+4\cdot B\right)}{const_1}
\end{cases}
$$

$$
\text{II)}\quad\begin{cases}
\frac{B_s\cdot C_f}{R^{**}}=const_2=\frac{(1-n)\cdot B\cdot C_f}{M^{2/3}\cdot R^*}\\
\Rightarrow\quad R^*=\frac{(1-n)\cdot B\cdot C_f}{const_2}\cdot M
\end{cases}
\quad. \tag{9}
$$

$$\Leftrightarrow$$

$$
\Rightarrow\quad\frac{\left(A\cdot(n-6)+4\cdot B\right)}{const_1}=4\cdot\frac{B\cdot C_f}{const_2}\,. \tag{10}
$$

Assuming that also the parameters C_f, A and B do not directly depend on the various masses Mi within the solutions for the Ψ_1,Ψ_2,Ψ_3, we have a connection of the eigenvalues A and B of the differential operators in equation (4) (first line). Most interestingly, in 6 dimensions equation (10) significantly simplifies (collapses more like) and gives us:

$$
\Rightarrow\quad C_f=\frac{const_2}{const_1}=\frac{\frac{(1-n)\cdot B\cdot C_f}{R^*}\cdot M}{\frac{(1-n)}{4\cdot R^*}\cdot M\cdot\left(A\cdot(n-6)+4\cdot B\right)}=\xrightarrow{n=6}=C_f\,. \tag{11}
$$

As before we can extract two equations for the determination of two of the three states Ψ_i out of the other:

$$C_f = -\frac{(\Psi_1+\Psi_2+\Psi_3)}{3}$$

$$\Rightarrow \mathrm{const}_1 = \frac{1}{3}\cdot(\Psi_1+\Psi_2+\Psi_3)^2 - \Psi_1\Psi_2 - \Psi_1\Psi_3 - \Psi_2\Psi_3 \, . \tag{12}$$

$$\Rightarrow \mathrm{const}_2 = \Psi_1\Psi_2\Psi_3 + \left(\frac{(\Psi_1+\Psi_2+\Psi_3)}{3}\right)^3$$

In contrast to the results posted before, where we either had a Klein-Gordon or a Dirac kind of approach, this time we have no need for an additional condition $\Delta f = 0$ as we had to demand it in the two previous posts about the Dirac case. We only need to demand that the function f has eigenvalue solutions A and B to the differential operators in equation (4) (first line) as follows $\Delta f = B\cdot f$ and $f_{,\alpha}g^{\alpha\beta}f_{,\beta} = A\cdot f^2$. This means, the moment these eigenvalues exist, we automatically have an explanation for the 3-generation problem of the elementary particles, namely simply through the fact that the Ricci scalar of a scale variated metric with the simple variation $G_{\alpha\beta}=F[f]*g_{\alpha\beta}=(f+C_f/M)*g_{\alpha\beta}$ forms an algebraic equation of third order and by solving this equation with respect to f and M we obtain 3 solutions for the combination of mass M and the quantum gravity function f.

It also should be pointed out that the combined demand for the existence of eigenvalue solutions to both operators $\Delta f = B\cdot f$ and $f_{,\alpha}g^{\alpha\beta}f_{,\beta} = A\cdot f^2$ in 4 dimensions would automatically favor octonions, which is to say the bigger brothers of Dirac's quaternions, as just THE obvious tool for the solution of the subsequent complete differential equation in (4) (first line).

However, as demonstrated in [2] sub-section "Getting rid of the Quaternions", there may be a way around these complex features and the subsequent cumbersome evaluations with the many restrictions they automatically bring with themselves. Thus, in [2] we will try to "Avoid the Octonions" and still find first order differential equations to eigenvalue equations with mixed differential operators $\Delta f = B\cdot f$ and $f_{,\alpha}g^{\alpha\beta}f_{,\beta} = A\cdot f^2$ leading to:

$$\begin{aligned} R^* &= \frac{(1-n)}{4\cdot\left(f+\frac{C_f}{M}\right)^3}\cdot\left((6-n)\cdot f_{,\alpha}g^{\alpha\beta}f_{,\beta} + 4\cdot\left(f+\frac{C_f}{M}\right)\cdot\Delta f\right) \\ &= f\cdot\frac{(1-n)}{4\cdot\left(f+\frac{C_f}{M}\right)^3}\cdot\left(A\cdot(6-n)\cdot f + 4\cdot B\cdot\left(f+\frac{C_f}{M}\right)\right) \end{aligned} \quad . \tag{13}$$

$$\xrightarrow{\text{via octonions ?}}$$

$$\Rightarrow \boxed{f_{,\alpha}g^{\alpha\beta}f_{,\beta} + \frac{4}{(6-n)}\cdot\left(f+\frac{C_f}{M}\right)\cdot\Delta f = A\cdot f^2 + \frac{4\cdot B}{(6-n)}\cdot f\cdot\left(f+\frac{C_f}{M}\right)}$$

More discussion shall be presented elsewhere (e.g. [2]).

References

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