The So-Called 3-Generations Problem of Elementary Particles
Here: The Particle at Rest and Some Interpretation

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The Three Generations of Particles, by Livia Schwarzer

Theory: The Ricci Scalar for a Scaled Metric

In [1, 2, 3] (and quite some other papers by this author) it was demonstrated how we can find a direct extraction of the Klein-Gordon, the Schrödinger and the Dirac equation from the Ricci scalar R^* of a modified metric of the kind $G_{\alpha\beta}=F[f]*g_{\alpha\beta}$. Namely: With a yet arbitrary scalar function F[f] the corresponding modified Ricci scalar R^* reads:

$$\begin{aligned} R^* &= \frac{1}{F[f]^3}\cdot\left(\begin{array}{c}\left(C_{N1}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2 - C_{N2}\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\overbrace{\left(\tilde{\nabla}_g f\right)^2}^{=f_{,\alpha}g^{\alpha\beta}f_{,\beta}} \\ -C_{N2}\cdot F[f]\cdot\frac{\partial F[f]}{\partial f}\cdot\Delta_g f\end{array}\right) \\ &\xrightarrow{n=4} = \frac{1}{F[f]^3}\cdot\left(\left(\frac{3}{2}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2 - 3\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\left(\tilde{\nabla}_g f\right)^2 - 3\cdot F[f]\cdot\frac{\partial F[f]}{\partial f}\cdot\Delta_g f\right)\end{aligned} \quad . \tag{1}$$

Thereby the C_{N1} and C_{N2} are constants depending on the number of dimensions n of the space-time being considered. For illustration we also gave the case n=4 (n=number of dimensions). Thereby f is to be understood as a function of the coordinates $f=f[x_0, x_1, x_2, \ldots]$. It was shown in [1 - 6] (especially [2]) that we can write:

$$\begin{aligned} C_{N1} &= -\frac{3}{2}+\frac{7}{4}\cdot n-\frac{1}{4}\cdot n^2, \\ C_{N2} &= n-1 \end{aligned} \tag{2}$$

Demanding certain conditions for the function F[f] and / or f then gives us Dirac or Klein-Gordon equations [1, 2, 3]. Thus, when demanding f to be a Laplace function, we obtain from (1):

$$\begin{aligned} R^* &= \frac{1}{F[f]^3}\cdot\left(\left(C_{N1}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2 - C_{N2}\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\overbrace{\left(\tilde{\nabla}_g f\right)^2}^{=f_{,\alpha}g^{\alpha\beta}f_{,\beta}}\right), \\ &\xrightarrow{n=4} = \frac{1}{F[f]^3}\cdot\left(\left(\frac{3}{2}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2 - 3\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\left(\tilde{\nabla}_g f\right)^2\right)\end{aligned} \tag{3}$$

which – so it was shown in [1, 2, 3] – gives the metric equivalent to the Dirac equation. It was also shown in [1, 2] how this gives the classical Dirac equation in flat space Minkowski metrics.

Finally – Klein Gordon + Dirac: A Most Simple Explanation for the 3-Generations Problem

This time we will not try to get rid of one of the two differential operators in equation (1), but assume them to have eigen solutions with eigenvalues A and B as shown in [2] or our earlier posts here (see "General F[f] - here Nabla and Laplace with Eigenvalues.pdf"). Now, just as in the most recent post, we apply the following simple linear form for F[f] via F[f]=(f+C_f/M). This gives us from equation (1), with time without the assumption of f being a Laplace solution, the following result:

$$\begin{aligned} R^* &= \frac{(1-n)}{4\cdot\left(f+\frac{C_f}{M}\right)^3}\cdot\left((6-n)\cdot f_{,\alpha}g^{\alpha\beta}f_{,\beta}+4\cdot\left(f+\frac{C_f}{M}\right)\cdot\Delta f\right) \\ &= f\cdot\frac{(1-n)}{4\cdot\left(f+\frac{C_f}{M}\right)^3}\cdot\left(A\cdot(6-n)\cdot f+4\cdot B\cdot\left(f+\frac{C_f}{M}\right)\right) \end{aligned}, \quad (4)$$

Now we follow the path outlined in the previous post and algebraically solve the equation above. In order to make things easier with respect to the 3-generations problem, we substitute as follows:

$$\Psi = M\cdot f;\ R^{**}=4\cdot\frac{R^*}{M};\ B_s=4\cdot(1-n)\cdot B;\ A_s=A\cdot(1-n)\cdot(n-6)$$

$$\begin{aligned} &\Rightarrow R^*=\frac{\Psi}{M}\cdot\frac{(1-n)}{4\cdot\left(\frac{\Psi}{M}+\frac{C_f}{M}\right)^3}\cdot\left(A\cdot(6-n)\cdot\frac{\Psi}{M}+4\cdot B\cdot\left(\frac{\Psi}{M}+\frac{C_f}{M}\right)\right) \quad , \quad (5)\\ &\Rightarrow 4\cdot R^*\cdot\left(\frac{\Psi}{M}+\frac{C_f}{M}\right)^3=\frac{\Psi}{M}\cdot(1-n)\cdot\left(A\cdot(6-n)\cdot\frac{\Psi}{M}+4\cdot B\cdot\left(\frac{\Psi}{M}+\frac{C_f}{M}\right)\right) \\ &\Rightarrow R^{**}\cdot(\Psi+C_f)^3=4\cdot\frac{R^*}{M}\cdot(\Psi+C_f)^3=\Psi\cdot\left(A_s\cdot\Psi+B_s\cdot(\Psi+C_f)\right) \end{aligned}$$

which gives – after expansion and division by R^{**} – from (4):

$$(C_f)^3-\frac{B_s\cdot C_f}{R^{**}}+\left(3\cdot(C_f)^2-\frac{A_s}{R^{**}}-\frac{B_s}{R^{**}}\right)\cdot\Psi+3\cdot C_f\cdot\Psi^2+\Psi^3=0, \quad (6)$$

Again, we point out that for here and now (for the reason of simplicity and brevity mainly) we do not need to care about potential inner vector characters of the mass, the function f and the operator terms. For the moment we just assume that this does not have any influence on the 3-generations problem we want to consider here. The proof for this can easily be obtained by applying $f=h_\alpha q^\alpha$ in all derivations below. It will not change the principle results with regards to the 3-generation or 3-masses problem.

Equation (6) is a polynomial of third order and it can have three solutions. The general solution to a three-order polynomial could be given via the following product form:

$$\begin{aligned} &(\Psi-\Psi_1)\cdot(\Psi-\Psi_2)\cdot(\Psi-\Psi_3)= \\ &\Psi^3-\Psi^2\cdot(\Psi_1+\Psi_2+\Psi_3)+\Psi\cdot(\Psi_1\Psi_2+\Psi_1\Psi_3+\Psi_2\Psi_3)-\Psi_1\Psi_2\Psi_3 \end{aligned}. \quad (7)$$

Comparing the latter with (6) gives us:

$$3\cdot C_f = -\left(\Psi_1+\Psi_2+\Psi_3\right)$$
$$3\cdot\left(C_f\right)^2-\frac{A_s}{R^{**}}-\frac{B_s}{R^{**}}=\Psi_1\Psi_2+\Psi_1\Psi_3+\Psi_2\Psi_3 . \qquad (8)$$
$$\left(C_f\right)^3-\frac{B_s\cdot C_f}{R^{**}}=-\Psi_1\Psi_2\Psi_3$$

Thus, we have obtained the three generations of quantum gravity solutions to the combined mass-times-f-function, given via $\Psi\equiv M\cdot f\equiv\frac{m\cdot c}{\hbar}\cdot f$, as functions or dependencies of the Ricci scalar R* of the quantum-gravity (just scalar) variated metric $G_{\alpha\beta}=F[f]*g_{\alpha\beta}$, the mass-values $M\equiv\frac{m\cdot c}{\hbar}$ and a constant C_f.

Please note: As the expressions $\Psi_1+\Psi_2+\Psi_3,\ \Psi_1\Psi_2+\Psi_1\Psi_3+\Psi_2\Psi_3,\ \Psi_1\Psi_2\Psi_3$ on the right-hand side in (8) should be just constants and thus, could not depend on the various masses M_i, we have to demand the Ricci-scalar R^* to be directly connected with the M_i via the two constants $const_1$ and $const_2$:

$$\text{I)}\quad\begin{cases}\dfrac{A_s}{R^{**}}+\dfrac{B_s}{R^{**}}=const_1=\dfrac{(1-n)}{4\cdot R^*}\cdot M\cdot\left(A\cdot(n-6)+4\cdot B\right)\\ \quad\Rightarrow\quad R^*=\dfrac{(1-n)}{4}\cdot M\cdot\dfrac{\left(A\cdot(n-6)+4\cdot B\right)}{const_1}\end{cases}$$
$$\text{II)}\quad\begin{cases}\dfrac{B_s\cdot C_f}{R^{**}}=const_2=\dfrac{(1-n)\cdot B\cdot C_f}{M^{2/3}\cdot R^*}\\ \quad\Rightarrow\quad R^*=\dfrac{(1-n)\cdot B\cdot C_f}{const_2}\cdot M\end{cases} . \qquad (9)$$
$$\Leftrightarrow$$
$$\Rightarrow\quad\frac{\left(A\cdot(n-6)+4\cdot B\right)}{const_1}=4\cdot\frac{B\cdot C_f}{const_2} . \qquad (10)$$

Assuming that also the parameters C_f, A and B do not directly depend on the various masses Mi within the solutions for the Ψ_1,Ψ_2,Ψ_3, we have a connection of the eigenvalues A and B of the differential operators in equation (4) (first line). Most interestingly, in 6 dimensions equation (10) significantly simplifies (collapses more like) and gives us:

$$\Rightarrow\quad C_f=\frac{const_2}{const_1}=\frac{\dfrac{(1-n)\cdot B\cdot C_f}{R^*}\cdot M}{\dfrac{(1-n)}{4\cdot R^*}\cdot M\cdot\left(A\cdot(n-6)+4\cdot B\right)}=\xrightarrow{n=6}=C_f . \qquad (11)$$

As before we can extract two equations for the determination of two of the three states Ψ_i out of the other:

$$C_f = -\frac{\left(\Psi_1+\Psi_2+\Psi_3\right)}{3}$$

$$\begin{aligned}&\Rightarrow \mathrm{const}_1 = \frac{1}{3}\cdot\left(\Psi_1+\Psi_2+\Psi_3\right)^2-\Psi_1\Psi_2-\Psi_1\Psi_3-\Psi_2\Psi_3 \, . \\ &\Rightarrow \mathrm{const}_2 = \Psi_1\Psi_2\Psi_3+\left(\frac{\left(\Psi_1+\Psi_2+\Psi_3\right)}{3}\right)^3\end{aligned} \tag{12}$$

In contrast to the results posted before, where we either had a Klein-Gordon or a Dirac kind of approach, this time we have no need for an additional condition $\Delta f = 0$ as we had to demand it in the two previous posts about the Dirac case. We only need to demand that the function f has eigenvalue solutions A and B to the differential operators in equation (4) (first line) as follows $\Delta f = B\cdot f$ and $f_{,\alpha}g^{\alpha\beta}f_{,\beta} = A\cdot f^2$. This means, the moment these eigenvalues exist, we automatically have an explanation for the 3-generation problem of the elementary particles, namely simply through the fact that the Ricci scalar of a scale variated metric with the simple variation $G_{\alpha\beta}=F[f]*g_{\alpha\beta}=(f+C_f/M)*g_{\alpha\beta}$ forms an algebraic equation of third order and by solving this equation with respect to f and M we obtain 3 solutions for the combination of mass M and the quantum gravity function f.

It also should be pointed out that the combined demand for the existence of eigenvalue solutions to both operators $\Delta f = B\cdot f$ and $f_{,\alpha}g^{\alpha\beta}f_{,\beta} = A\cdot f^2$ in 4 dimensions would automatically favor octonions, which is to say the bigger brothers of Dirac's quaternions, as just THE obvious tool for the solution of the subsequent complete differential equation in (4) (first line).

However, as demonstrated in [2] sub-section "Getting rid of the Quaternions", there may be a way around these complex features and the subsequent cumbersome evaluations with the many restrictions they automatically bring with themselves. Thus, in [2] we will try to "Avoid the Octonions" and still find first order differential equations to eigenvalue equations with mixed differential operators $\Delta f = B\cdot f$ and $f_{,\alpha}g^{\alpha\beta}f_{,\beta} = A\cdot f^2$ leading to:

$$\begin{aligned}R^* &= \frac{(1-n)}{4\cdot\left(f+\frac{C_f}{M}\right)^3}\cdot\left((6-n)\cdot f_{,\alpha}g^{\alpha\beta}f_{,\beta}+4\cdot\left(f+\frac{C_f}{M}\right)\cdot\Delta f\right)\\ &= f\cdot\frac{(1-n)}{4\cdot\left(f+\frac{C_f}{M}\right)^3}\cdot\left(A\cdot(6-n)\cdot f+4\cdot B\cdot\left(f+\frac{C_f}{M}\right)\right) \, .\\ &\xrightarrow{\text{via octonions ?}}\\ \Rightarrow &\boxed{f_{,\alpha}g^{\alpha\beta}f_{,\beta}+\frac{4}{(6-n)}\cdot\left(f+\frac{C_f}{M}\right)\cdot\Delta f = A\cdot f^2+\frac{4\cdot B}{(6-n)}\cdot f\cdot\left(f+\frac{C_f}{M}\right)}\end{aligned} \tag{13}$$

More discussion shall be presented elsewhere (e.g. [2] and next section).

Interpretation and the Particle at Rest

Finding a general solution to (13) (last line) seems to be rather difficult even in the simplest cases. So, we were not able to find a solution for the so-called particle at rest problem with f=f[t]. Only in the case of $C_f=0$, we found the solution:

$$f[t] = C_{t1} \cdot \cos\left[\frac{\sqrt{(n-2)\cdot(A\cdot(6-n)-4B)}}{4}\cdot(t+C_{t2})\right]^{\frac{4}{-2+n}} . \tag{14}$$

These are the usual oscillating solutions for positive values under the square root. Thus, the solution, even though we did not resort to the Dirac factorization, already is not very different from the classical result (c.f. [7] and "particle at rest solution" in the text books). The oscillations are completely symmetric in positive and negative direction of time and thus, there is no asymmetry between matter and antimatter. We guess, however, that things are changing here the moment we allow for $C_f\neq0$.

Could this lead then to some asymmetry, potentially also explaining the matter anti-matter discrepancy of our universe?

Summing up now what we have found we can state the following:

A) **When introducing a scale-variated metric of the kind $G_{\alpha\beta}=F[f]*g_{\alpha\beta}$ we obtain a Ricci-scalar R^* which contains the function f in such a form, that we can extract both, Dirac and Klein-Gordon equations. As the origin is completely geometry, respectively of 100% metric character, we consider our approach as a quantum gravity one and, thus, consequently, the function F[f], respectively f as a quantum gravity wave function.**

B) **Adding dimensions with special dependencies to the "main (space-time) dimensions", which we here saw AND treated as entanglement, brings all forms of inertia, spin and stuff, even when demanding the total scalar curvature R^* to vanish.**

C) **Not demanding the total scalar curvature to vanish and assuming that there are eigen-solutions to the differential equation of the Ricci-scalar R^* for the function f, leads to algebraic equations allowing for 3 different solutions to the combined product of mass and the quantum gravity wave function f. We think that this is the answer to the riddle of the 3 generations of elementary particles.**

D) **The interesting point is here, that all classical quantum effects, including an obviously rather potent solution to the 3-generaion problem, arise from a simple scale factor F[f] to the metric $g_{\alpha\beta}$.**

E) **We also saw, even though we were not able - so far anyway - to completely solve the subsequent complex eigen equation for the function f resulting from the adapted Ricci scalar, that the so-called "particle at rest" sports the usual oscillations in time as we already know them from the classical Dirac theory [7].**

F) **From this automatically also follows (among other things, which should be discussed later on) that particle masses are apparently just static (or quasi-static) scale oscillations of space and time. In other words, where space-time vibrates there is mass and where space-time vibrates in a stable manner, there are stable particles with mass.**

References

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