Towards a Solution to the So-Called 3-Generations Problem of Elementary Particles - Starting Point this time: The Metric Dirac Equation Or: Why are There 3 masses for Charged Leptons, Neutrons and Quarks?

Norbert Schwarzer, www.worldformulaapps.com

contact via: www.worldformulaapps.com/#contact-module

The Three Generations of Particles, by Livia Schwarzer

Theory: The Ricci Scalar for a Scaled Metric

In [1, 2, 3] (and quite some other papers by this author) it was demonstrated how we can find a direct extraction of the Klein-Gordon, the Schrödinger and the Dirac equation from the Ricci scalar R^* of a modified metric of the kind $G_{\alpha\beta}=F[f]*g_{\alpha\beta}$. Namely: With a yet arbitrary scalar function F[f] the corresponding modified Ricci scalar R^* reads:

$$\begin{aligned} R^* &= \frac{1}{F[f]^3}\cdot\left(\begin{array}{c}\left(C_{N1}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2 - C_{N2}\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\overbrace{\left(\tilde{\nabla}_g f\right)^2}^{=f_{,\alpha}g^{\alpha\beta}f_{,\beta}} \\ -C_{N2}\cdot F[f]\cdot\frac{\partial F[f]}{\partial f}\cdot\Delta_g f\end{array}\right) \\ &\xrightarrow{n=4} = \frac{1}{F[f]^3}\cdot\left(\left(\frac{3}{2}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2 - 3\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\left(\tilde{\nabla}_g f\right)^2 - 3\cdot F[f]\cdot\frac{\partial F[f]}{\partial f}\cdot\Delta_g f\right) \end{aligned} \quad . \qquad (1)$$

Thereby the C_{N1} and C_{N2} are constants depending on the number of dimensions n of the space-time being considered. For illustration we also gave the case n=4 (n=number of dimensions). Thereby f is to be understood as a function of the coordinates $f=f[x_0, x_1, x_2, \ldots]$. It was shown in [1 - 6] (especially [2]) that we can write:

$$\begin{aligned} C_{N1} &= -\frac{3}{2}+\frac{7}{4}\cdot n-\frac{1}{4}\cdot n^2 \\ C_{N2} &= n-1 \end{aligned} , \qquad (2)$$

Demanding certain conditions for the function F[f] and / or f then gives us Dirac or Klein-Gordon equations [1, 2, 3]. Thus, when demanding f to be a Laplace function, we obtain from (1):

$$\begin{aligned} R^* &= \frac{1}{F[f]^3}\cdot\left(\left(C_{N1}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2 - C_{N2}\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\overbrace{\left(\tilde{\nabla}_g f\right)^2}^{=f_{,\alpha}g^{\alpha\beta}f_{,\beta}}\right) \\ &\xrightarrow{n=4} = \frac{1}{F[f]^3}\cdot\left(\left(\frac{3}{2}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2 - 3\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\left(\tilde{\nabla}_g f\right)^2\right) \end{aligned} , \qquad (3)$$

which – so it was shown in [1, 2, 3] – gives the metric equivalent to the Dirac equation. It was also shown in [1, 2] how this gives the classical Dirac equation in flat space Minkowski metrics.

Towards an Explanation for the 3-Generations Problem (?) with Respect to the Metric Dirac Equation [1, 2]

There is a variety of options to derive discrete masses for only 3 generations in the Dirac case.

A) Via the classical Dirac path by extracting the square root from the Klein-Gordon operator, thereby applying quaternions. Having previously shown that there is a 3-generation solution for the Klein-Gordon equation, the same then also follows for the subsequent Dirac equation. Thus, we can consider the job of following this option already done.

B) Via an $\Delta_g f = 0$ approach as used here for the derivation of (3) and a suitable choice for the function F[f].

C) Via a suitable assumption for $\frac{\partial F[f]}{\partial f} \simeq 0$ leading to an approach with a scalar field somewhat similar to the Higgs mechanics.

D) …

All these options split up into more possibilities when distinguishing the way of root extraction (Dirac-like or via vectors as recently shown here (see "Dirac getting rid of the quaternions.pdf").

Here, in order to give at least one example, we only concentrate on the fairly simple option B.

We apply the following exponential form for F[f] via F[f]=(f+C_f/M)^q. This gives us from equation (1) in the case of q=4:

$$R^* = \frac{4 \cdot (n-3)(1-n)}{\left(f + \frac{C_f}{M}\right)^6} \cdot \overbrace{\left(\tilde{\nabla}_g f\right)^2}^{=f_{,\alpha} g^{\alpha\beta} f_{,\beta}}, \tag{4}$$

We have learned from [1, 2] that (3) or – just simpler - (4) is just the square of the metric Dirac equation of which the classical Dirac equation can easily be derived when moving towards Minkowski coordinates (again the reader is referred to [1, 2] or one of our more recent posts with respect to the mathematical details – see document "Direct Extraction of the Dirac Equation from the Metric Tensor.pdf" also posted here recently). Thus, regarding the 3-generations problem of elementary particles, we simply can proceed with the square form (4) in order to show that also the metric Dirac equation could result in the 3 generations. Subsequent extraction of the square root (via Quaternions or vectors as also recently posted here) will not change the results. This can easily be shown by remembering that f definitively is a scalar (and thus, has to be treated as one in the following calculation), however, that when moving towards the metric Dirac equation we simply have to assume this scalar to be decomposable as $f = h_\alpha q^\alpha$ (see document "Direct Extraction of the Dirac Equation from the Metric Tensor.pdf" also posted here recently).

We know that the operator square of the classical Dirac equation gives the classical Klein-Gordon equation. It always is of the kind:

$$-M^2 f + \Delta_g f = \left(-M^2 + \Delta_g\right) f = 0$$
$$\text{with:} \quad M^2 \equiv \frac{m^2 \cdot c^2}{\hbar^2} \tag{5}$$

(c… speed of light in vacuum, m… rest mess, $\hbar$ …reduced Planck constant).

Because these linear forms work so well, we conclude that – in this universe – with respect to the mass term M² there has to exist a linear form to (4), which solutions have to be at least temporarily stable. It should read:

$$M^2 \cdot f^2 = \left(\tilde{\nabla}_g f\right)^2 \Rightarrow M \cdot f = \left(\tilde{\nabla}_g f\right). \tag{6}$$

Please note that for here and now (for the reason of simplicity and brevity mainly) we did not care about potential vector characters of the mass and operator terms. One can prove that this does in fact not have any influence on the 3-generations problem we want to consider here. Comparing the last equation (6) with the square of the metric Dirac equation of our choice for F[f]]=(f+C_f/M)^q tells us, that finding a squared linearization for the expression $\frac{R^*}{4\cdot(n\text{-}3)(1-n)}\cdot\left(f+\frac{C_f}{M}\right)^6$ with respect to the function f, would immediately give us the classical Dirac quantum equation from a completely metric origin. Thus, we have to look for possible – potentially approximated - solutions to the equation:

$$\begin{aligned} M^2 \cdot f^2 \equiv \frac{m^2 \cdot c^2}{\hbar^2} \cdot f^2 &= \frac{R^*}{4\cdot(n\text{-}3)(1-n)}\cdot\left(f+\frac{C_f}{M}\right)^6 \\ \Rightarrow M \cdot f &= \frac{\sqrt{R^*}}{2\cdot\sqrt{(n\text{-}3)(1-n)}}\cdot\left(f+\frac{C_f}{M}\right)^3 \end{aligned}. \tag{7}$$

This, however, is a polynomial of third order and it can have three solutions.

A bit of reshaping the last equation is going to help us to realize how this will possibly also solve the 3-generations mass problem:

$$\begin{aligned} \Psi \equiv M \cdot f \equiv \frac{m \cdot c}{\hbar} \cdot f &= \frac{\sqrt{R^*}}{2\cdot\sqrt{(n\text{-}3)(1-n)}}\cdot\left(f+\frac{C_f}{M}\right)^3 = \frac{\sqrt{R^*}}{2\cdot M^3\cdot\sqrt{(n\text{-}3)(1-n)}}\cdot\left(\Psi + C_f\right)^3 \\ &\xrightarrow{R^{**}=\frac{\sqrt{R^*}}{2\cdot M^3\cdot\sqrt{(n\text{-}3)(1-n)}};\quad R^{***}=C_f} R^{**}\cdot\left(\Psi+R^{***}\right)^3 - \Psi = 0 \\ &= \left(\Psi+R^{***}\right)^3 - \frac{\Psi}{R^{**}} = \left(R^{***}\right)^3 - \frac{\Psi}{R^{**}} + 3\cdot\left(R^{***}\right)^2\cdot\Psi + 3\cdot R^{***}\cdot\Psi^2 + \Psi^3 \end{aligned}. \tag{8}$$

The general solution to a three-order polynomial could be given via the following product form:

$$\begin{aligned} &(\Psi-\Psi_1)\cdot(\Psi-\Psi_2)\cdot(\Psi-\Psi_3) = \\ &\Psi^3 - \Psi^2\cdot(\Psi_1+\Psi_2+\Psi_3) + \Psi\cdot(\Psi_1\Psi_2+\Psi_1\Psi_3+\Psi_2\Psi_3) - \Psi_1\Psi_2\Psi_3 \,. \\ &\left(R^{***}\right)^3 - \frac{\Psi}{R^{**}} + 3\cdot\left(R^{***}\right)^2\cdot\Psi + 3\cdot R^{***}\cdot\Psi^2 + \Psi^3 \end{aligned} \tag{9}$$

Comparing the latter with the last line in (8) gives us:

$$\begin{aligned} 3\cdot R^{***} = 3\cdot C_f &= -(\Psi_1+\Psi_2+\Psi_3) \\ 3\cdot\left(R^{***}\right)^2 - \frac{1}{R^{**}} = 3\cdot(C_f)^2 - \frac{2\cdot M^3\cdot\sqrt{(n\text{-}3)(1-n)}}{\sqrt{R^*}} &= \Psi_1\Psi_2+\Psi_1\Psi_3+\Psi_2\Psi_3 \,. \\ (C_f)^3 &= -\Psi_1\Psi_2\Psi_3 \end{aligned} \tag{10}$$

Thus, we have obtained the three generations of quantum gravity solutions to the combined mass-times-f-function, given via $\Psi \equiv \mathrm{M} \cdot \mathrm{f} \equiv \frac{\mathrm{m} \cdot \mathrm{c}}{\hbar} \cdot \mathrm{f}$, as functions or dependencies of the Ricci scalar R* of the quantum-gravity (just scalar) variated metric $G_{\alpha\beta}$=F[f]*$g_{\alpha\beta}$, the mass-values $\mathrm{M} \equiv \frac{\mathrm{m} \cdot \mathrm{c}}{\hbar}$ and a constant C_f.

Please note: As the expression $\Psi_1 \Psi_2 + \Psi_1 \Psi_3 + \Psi_2 \Psi_3$ on the right-hand side in the second line in (10) should be just a constant and thus, could not depend on the various masses M_i, we have to demand the Ricci-scalar R^* to be directly connected with the M_i via a constant const:

$$\frac{2 \cdot \mathrm{M}^3 \cdot \sqrt{(n\text{-}3)(1-n)}}{\sqrt{\mathrm{R}^*}} = \sqrt{\mathrm{const}} \quad \Rightarrow \quad \mathrm{R}^* = 4 \cdot \frac{(n\text{-}3)(1-n) \cdot \mathrm{M}^6}{\mathrm{const}} . \tag{11}$$

This gives two equations for the extraction of two of the three states Ψ_i out of the other:

$$\begin{aligned} \mathrm{R}^{***} = \mathrm{C}_\mathrm{f} &= -\frac{\Psi_1 + \Psi_2 + \Psi_3}{3} \\ \Rightarrow \frac{1}{3} \cdot \left(\Psi_1 + \Psi_2 + \Psi_3\right)^2 &- \frac{2 \cdot \mathrm{M}^3 \cdot \sqrt{(n\text{-}3)(1-n)}}{\sqrt{\mathrm{R}^*}} \\ = \frac{1}{3} \cdot \left(\Psi_1 + \Psi_2 + \Psi_3\right)^2 - \mathrm{const} &= \Psi_1 \Psi_2 + \Psi_1 \Psi_3 + \Psi_2 \Psi_3 \\ \Rightarrow \left(\frac{\Psi_1 + \Psi_2 + \Psi_3}{3}\right)^3 &= \Psi_1 \Psi_2 \Psi_3 \end{aligned} . \tag{12}$$

As before (see our recent post, where we considered the 3-generation problem in connection with the Klein-Gordon equation), we find that with a vanishing constant const (the curvature term) we would obtain one solution, namely:

$$\Psi_1 = \Psi_2 = \Psi_3 . \tag{13}$$

We also find that with a vanishing constant C_f (the shift of the function f with the value F[f]=f=0 moving away from the origin), we would only obtain the solution

$$\Psi_2 = \Psi_3 = \frac{1}{2}\left(-\Psi_1 \pm \sqrt{4 \cdot \mathrm{const} - 3 \cdot \Psi_1^2}\right). \tag{14}$$

More discussion and the other options shall be presented elsewhere (e.g. [2]).

References

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