Towards a Solution to the So-Called 3-Generations Problem of Elementary Particles - Starting Point this time: The Metric Dirac Equation – Part II Or: Why are There 3 masses for Charged Leptons, Neutrons and Quarks?

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The Three Generations of Particles, by Livia Schwarzer

Theory: The Ricci Scalar for a Scaled Metric

In [1, 2, 3] (and quite some other papers by this author) it was demonstrated how we can find a direct extraction of the Klein-Gordon, the Schrödinger and the Dirac equation from the Ricci scalar R^* of a modified metric of the kind $G_{\alpha\beta}=F[f]*g_{\alpha\beta}$. Namely: With a yet arbitrary scalar function F[f] the corresponding modified Ricci scalar R^* reads:

$$R^* = \frac{1}{F[f]^3}\cdot\left(\begin{array}{c}\left(C_{N1}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2 - C_{N2}\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\overbrace{\left(\tilde{\nabla}_g f\right)^2}^{=f_{,\alpha}g^{\alpha\beta}f_{,\beta}} \\ -C_{N2}\cdot F[f]\cdot\frac{\partial F[f]}{\partial f}\cdot\Delta_g f\end{array}\right) . \quad (1)$$

$$\xrightarrow{n=4} = \frac{1}{F[f]^3}\cdot\left(\left(\frac{3}{2}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2 - 3\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\left(\tilde{\nabla}_g f\right)^2 - 3\cdot F[f]\cdot\frac{\partial F[f]}{\partial f}\cdot\Delta_g f\right)$$

Thereby the C_{N1} and C_{N2} are constants depending on the number of dimensions n of the space-time being considered. For illustration we also gave the case n=4 (n=number of dimensions). Thereby f is to be understood as a function of the coordinates $f=f[x_0, x_1, x_2, \ldots]$. It was shown in [1 - 6] (especially [2]) that we can write:

$$\begin{array}{c} C_{N1} = -\frac{3}{2}+\frac{7}{4}\cdot n-\frac{1}{4}\cdot n^2 \\ C_{N2} = n-1 \end{array}, \quad (2)$$

Demanding certain conditions for the function F[f] and / or f then gives us Dirac or Klein-Gordon equations [1, 2, 3]. Thus, when demanding f to be a Laplace function, we obtain from (1):

$$\begin{array}{l} R^* = \frac{1}{F[f]^3}\cdot\left(\left(C_{N1}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2 - C_{N2}\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\overbrace{\left(\tilde{\nabla}_g f\right)^2}^{=f_{,\alpha}g^{\alpha\beta}f_{,\beta}}\right) \\ \xrightarrow{n=4} = \frac{1}{F[f]^3}\cdot\left(\left(\frac{3}{2}\cdot\left(\frac{\partial F[f]}{\partial f}\right)^2 - 3\cdot F[f]\cdot\frac{\partial^2 F[f]}{\partial f^2}\right)\cdot\left(\tilde{\nabla}_g f\right)^2\right) \end{array}, \quad (3)$$

which – so it was shown in [1, 2, 3] – gives the metric equivalent to the Dirac equation. It was also shown in [1, 2] how this gives the classical Dirac equation in flat space Minkowski metrics.

Towards an Explanation for the 3-Generations Problem (?) with Respect to the Metric Dirac Equation [1, 2]

There is a variety of options to derive discrete masses for only 3 generations in the Dirac case.

A) Via the classical Dirac path by extracting the square root from the Klein-Gordon operator, thereby applying quaternions. Having previously shown that there is a 3-generation solution for the Klein-Gordon equation, the same then also follows for the subsequent Dirac equation. Thus, we can consider the job of following this option already done.

B) Via an $\Delta_g f = 0$ approach as used here for the derivation of (3) and a suitable choice for the function F[f].

C) Via a suitable assumption for $\frac{\partial F[f]}{\partial f} \simeq 0$ leading to an approach with a scalar field somewhat similar to the Higgs mechanics.

D) …

All these options split up into more possibilities when distinguishing the way of root extraction (Dirac-like or via vectors as recently shown here (see "Dirac getting rid of the quaternions.pdf").

Here, in order to give at least one example, we only concentrate on the fairly simple option B.

Now we apply the following most simple linear form for F[f] via F[f]=(f+C_f/M). This gives us from equation (1), together with the assumption of f being a Laplace solution, the following result:

$$R^* = \frac{(n-6)(1-n)}{4 \cdot \left(f + \frac{C_f}{M}\right)^3} \cdot \overbrace{\left(\tilde{\nabla}_g f\right)^2}^{=f_{,\alpha} g^{\alpha\beta} f_{,\beta}}, \tag{4}$$

We have learned from [1, 2] that (3) or – even simpler here (4) - is just the square of the metric Dirac equation of which the classical Dirac equation can easily be derived when moving towards Minkowski coordinates (again the reader is referred to [1] or the sections above with respect to the mathematical details – especially see [2] sub-section "Dirac in the Metric Picture and its Connection to the Classical Quaternion Form" above). Thus, regarding the 3-generations problem of elementary particles, we simply can proceed with the square form (4) in order to show that also the metric Dirac equation could result in the 3 generations. Subsequent extraction of the square root (via Quaternions or vectors as also given above in here) will not change the results. With respect to the "square character" of (3) and now also (4), however, we need to remember that this only holds in the case of the classical approximation (c.f. [2] section "Dirac in the Metric Picture and its Connection to the Classical Quaternion Form"), where we assumed the left-hand side of the metric Dirac equation with the curvature term as derived from with the assumption of f being a Laplace function:

$$\begin{aligned}
\frac{F[f]^2 \cdot R^*}{C_{N2} \cdot \frac{\partial^2 F[f]}{\partial f^2}} &\simeq h_\lambda q^\lambda \mathbf{M} \cdot \mathbf{M} h_\lambda q^\lambda = -h_{\lambda,\alpha} q^\lambda g^{\alpha\beta} h_{\lambda,\beta} q^\lambda \\
&\xrightarrow{g^{\alpha\beta}=\mathbf{e}^\alpha \cdot \mathbf{e}^\beta} = -h_{\lambda,\alpha} q^\lambda \mathbf{e}^\alpha \cdot \mathbf{e}^\beta h_{\lambda,\beta} q^\lambda \qquad , \\
&\Rightarrow \mp h_\lambda q^\lambda \mathbf{M} = i \cdot h_{\lambda,\alpha} q^\lambda \mathbf{e}^\alpha \\
&\Rightarrow 0 = i \cdot h_{\lambda,\alpha} \mathbf{e}^\alpha \pm h_\lambda \mathbf{M}
\end{aligned} \tag{5}$$

(first line) to be of square character in f. We already saw above, that this way the possible explanation for the 3-generation problem disappears (simply because the power-3 term of f has been approximated / classically postulated away). Thus, here now, we want to avoid this approximation and as long as we stick to the scalar f, things should stay simple and mathematically feasible.

Instead now of starting with the assumption that the linear appearance of f in the classical Klein-Gordon equation:

$$-M^2 f + \Delta_g f = \left(-M^2 + \Delta_g\right) f = 0 \quad \text{with:} \quad M^2 \equiv \frac{m^2 \cdot c^2}{\hbar^2} \tag{6}$$

(c… speed of light in vacuum, m… rest mess, $\hbar$ …reduced Planck constant) is kind of a "natural law", we now ignore this classical postulation of linearity, completely stick to the metric results, we derived so far and only assume – for simplicity - that we can find f^2-type eigen solutions to the operator term $f_{,\alpha} g^{\alpha\beta} f_{,\beta}$ in (3) and (4). For the latter the equation would then read:

$$R^* = \frac{(n\text{-}6)(1-n)}{4\cdot\left(f+\frac{C_f}{M}\right)^3}\cdot\overbrace{\left(\tilde{\nabla}_g f\right)^2}^{=f_{,\alpha}g^{\alpha\beta}f_{,\beta}} = \frac{(n\text{-}6)(1-n)}{4\cdot\left(f+\frac{C_f}{M}\right)^3}\cdot C_D^2\cdot M^2\cdot f^2 ,$$
$$\Rightarrow R^*\cdot\frac{4\cdot\left(f+\frac{C_f}{M}\right)^3}{(n\text{-}6)(1-n)} = C_D^2\cdot M^2\cdot f^2 \xrightarrow{\Psi^2\equiv M^2\cdot f^2} R^*\cdot\frac{4\cdot\left(\frac{\Psi}{M}+\frac{C_f}{M}\right)^3}{(n\text{-}6)(1-n)} = C_D^2\cdot\Psi^2 \tag{7}$$

Again, we point out that for here and now (for the reason of simplicity and brevity mainly) we do not need to care about potential inner vector characters of the mass, the function f and the operator terms. For the moment we just assume that this does not have any influence on the 3-generations problem we want to consider here. The proof for this can easily be obtained by applying $f = h_\alpha q^\alpha$ in all derivations below. It will not change the principle results with regards to the 3-generation or 3-masses problem.

Reshaping of (7) leads to:

$$\frac{R^*}{M^3}\cdot\frac{4\cdot\left(\Psi+C_f\right)^3}{(n\text{-}6)(1-n)} \xrightarrow{R^{**}\equiv\frac{R^*}{M^3}\cdot\frac{4}{(n\text{-}6)(1-n)}} = R^{**}\cdot\left(\Psi+C_f\right)^3 = C_D^2\cdot\Psi^2 , \tag{8}$$

This, again, is a polynomial of third order and it can have three solutions. Expansion helps us to realize how this will possibly also solve the 3-generations mass problem:

$$R^{**}\cdot\left(\Psi+C_f\right)^3 - C_D^2\cdot\Psi^2 = 0$$
$$\Rightarrow \left(C_f\right)^3 - C_D^2\cdot\frac{\Psi^2}{R^{**}} + 3\cdot\left(C_f\right)^2\cdot\Psi + 3\cdot C_f\cdot\Psi^2 + \Psi^3 = 0 . \tag{9}$$

The general solution to a three-order polynomial could be given via the following product form:

$$\left(\Psi-\Psi_1\right)\cdot\left(\Psi-\Psi_2\right)\cdot\left(\Psi-\Psi_3\right) =$$
$$\Psi^3 - \Psi^2\cdot\left(\Psi_1+\Psi_2+\Psi_3\right) + \Psi\cdot\left(\Psi_1\Psi_2+\Psi_1\Psi_3+\Psi_2\Psi_3\right) - \Psi_1\Psi_2\Psi_3 . \tag{10}$$

Comparing the latter with the last line in (9):

$$\left(C_f\right)^3 - C_D^2 \cdot \frac{\Psi^2}{R^{**}} + 3\cdot\left(C_f\right)^2 \cdot \Psi + 3\cdot C_f \cdot \Psi^2 + \Psi^3 = 0 . \tag{11}$$

gives us:

$$\begin{aligned} 3\cdot C_f - \frac{C_D^2}{R^{**}} &= -\left(\Psi_1 + \Psi_2 + \Psi_3\right) \\ 3\cdot\left(C_f\right)^2 &= \Psi_1\Psi_2 + \Psi_1\Psi_3 + \Psi_2\Psi_3 . \\ \left(C_f\right)^3 &= -\Psi_1\Psi_2\Psi_3 \end{aligned} \tag{12}$$

Thus, we have obtained the three generations of quantum gravity solutions to the combined mass-times-f-function, given via $\Psi \equiv M \cdot f \equiv \frac{m\cdot c}{\hbar}\cdot f$, as functions or dependencies of the Ricci scalar R* of the quantum-gravity (just scalar) variated metric $G_{\alpha\beta}=F[f]*g_{\alpha\beta}$, the mass-values $M \equiv \frac{m\cdot c}{\hbar}$ and a constant C_f.

Please note: As the expression $\Psi_1 + \Psi_2 + \Psi_3$ on the right-hand side in the first line in (12) should be just a constant and thus, could not depend on the various masses M_i, we have to demand the Ricci-scalar R^* to be directly connected with the M_i via a constant const:

$$\frac{C_D^2}{R^{**}} = \text{const} = \frac{C_D^2}{\frac{R^*}{M^3}\cdot\frac{4}{(n\text{-}6)(1-n)}} \quad \Rightarrow \quad R^* = \frac{C_D^2 \cdot M^3 \cdot (n\text{-}6)(1-n)}{4\cdot \text{const}} . \tag{13}$$

This gives two equations for the extraction of two of the three states Ψ_i out of the other:

$$\begin{aligned} C_f &= -\sqrt[3]{\Psi_1\Psi_2\Psi_3} \\ \Rightarrow \frac{C_D^2}{\frac{R^*}{M^3}\cdot\frac{4}{(n\text{-}6)(1-n)}} &= \Psi_1 + \Psi_2 + \Psi_3 - 3\cdot\sqrt[3]{\Psi_1\Psi_2\Psi_3} . \\ \Rightarrow 0 &= \Psi_1\Psi_2 + \Psi_1\Psi_3 + \Psi_2\Psi_3 - 3\cdot\left(\sqrt[3]{\Psi_1\Psi_2\Psi_3}\right)^2 \end{aligned} \tag{14}$$

We find that with a vanishing constant C_f (the shift of the function f with the value f=0 moving away from the origin), we would only obtain one non-trivial solution, namely
$\Psi_1 = \text{const}, \Psi_2 = \Psi_3 = 0; \Psi_2 = \text{const}, \Psi_1 = \Psi_3 = 0; \Psi_3 = \text{const}, \Psi_2 = \Psi_1 = 0$.

For a vanishing const we find equality of all Ψ_i with $\Psi_i = \frac{1}{3}\cdot C_f$.

The fact that we do not consider this a full solution to the 3-generation mass problem lays in the fact that we had to assume the existence of eigen-solutions to the operator $f_{,\alpha}g^{\alpha\beta}f_{\beta}$ in addition to the fact that f also has to be a Laplace function. It needs to be demonstrated that such solutions exist and that they have the right behavior. We will leave this task for later.

More discussion and the other options shall be presented elsewhere (e.g. [2]).

References

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